MA/STAT 41600 **Sample Exam 1**

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Name: KEY	

Instructions:

- All answers must be written clearly.
- You may use a calculator, but you must show all your work in order to receive credit.
- Be sure to erase or cross out any work that you do not want graded.
- If you need extra space, you may use the back sides of the exam pages (if you do, please write me a note so that I know where to look).
- You must include all work to receive full credit.

Question:	1	2	3	4	5	6	Total
Points:	0	0	0	0	0	0	0
Score:							

1. A certain state has license plates showing three numbers (0 through 9) and three letters (A through Z). How many different license plates are possible:

(a) If the numbers must come before the letters?

$$\underset{\#1}{\underline{10}} \cdot \underset{\#2}{\underline{10}} \cdot \underset{\#3}{\underline{10}} \cdot \underset{\text{Letter 1}}{\underline{26}} \cdot \underset{\text{Letter 2}}{\underline{26}} \cdot \underset{\text{Letter 3}}{\underline{26}}$$

$$= \boxed{10^3 \cdot 26^3}$$

(b) If there is no restriction on where the letters and numbers appear?

$$\binom{6}{3}\binom{3}{3} \cdot 10^3 \times 26^3 = \boxed{\binom{6}{3} 10^3 26^3}$$

Decide on which 3 spots to put in the Letters

the remaining 3 must go in the remaining 3 spots

2. Consider a standard deck of 52 cards.

   (a) A gin hand consists of 10 cards from a standard deck of 52 cards. Find the probability that a gin hand has all 10 cards of the same suit.

   (b) Find the probability that a gin hand has a three pair. (e.g. $aabbccdefg$)

a)

pick a suit

pick 10 cards of the chosen suit

$$\frac{\underbrace{\binom{4}{1}}_{\text{pick a suit}}\underbrace{\binom{13}{10}}_{\text{pick 10 cards of the chosen suit}}}{\binom{52}{10}}$$

choose suits of the pairs

order doesn't matter

1st pair

2nd pair

3rd pair

pick remaining ranks

pick the 4 singles

$$\frac{\binom{13}{3}\binom{4}{2}\binom{4}{2}\binom{4}{2}\binom{10}{4}\binom{4}{1}^4}{\binom{52}{10}}$$

3. An urn contains 6 red, 4 blue, 8 green and 2 yellow balls. If a set of 4 balls is randomly selected (no replacement), what is the probability that each of the balls will be

   (a) The same color?

   (b) Of different colors?

6R 4B 8G 2Y

a)

Red Blue Green

$$\frac{\binom{6}{4}+\binom{4}{4}+\binom{8}{4}}{\binom{20}{4}}$$

$= .0178$

b.)

$$\frac{\binom{6}{1}\binom{4}{1}\binom{8}{1}\binom{2}{1}}{\binom{20}{4}}$$

$= .0793$

4. Independent flips of a coin that lands on heads with probability p are made. What is the probability that

(a) the first 10 outcomes are tails?

We have $P(H) = p$ and $P(T) = 1-p$. Independent flips imply that

$$P(T_1 T_2 \cdots T_{10}) = P(T_1) P(T_2) \cdots P(T_{10})$$
$$= (1-p)^{10}$$

(b) the first 3 outcomes are heads?

Similarly

$$P(H_1 H_2 H_3) = p \cdot p \cdot p = p^3$$

(c) there are at least 1 heads in the first 10 outcomes?

E = at least 1 heads in 10 outcomes

E^c = no heads in first 10 trials

= all 10 outcomes are heads

$$\mathbb{P}(E) = 1 - P(E^c) = \boxed{1 - (1-p)^{10}}$$

5. A local college student goes to a bar 7 nights a week: 3 of the nights at bar A, 2 of the nights at bar B, and 2 of the nights at bar C. He'll get a girl's number 99 percent of the time at bar A, 96 percent of the time at bar B , and only 85 percent of the time at bar C.

(a) On a random night of the week, what is the probability that he gets a girl's number?

$\mathbb{P}(A) = \frac{3}{7}$, $\mathbb{P}(B) = \frac{2}{7}$, $\mathbb{P}(C) = \frac{2}{7}$,

$N = \{\text{Gets a number}\}$

$\mathbb{P}(N) = \mathbb{P}(N \mid A)\mathbb{P}(A) + \mathbb{P}(N \mid B)\mathbb{P}(B) + \mathbb{P}(N \mid C)\mathbb{P}(C)$

$= (.99)\frac{3}{7} + (.96)\frac{2}{7} + (.85)\frac{2}{7}$

$= .9414$

(b) Given that he doesn't get a girl's number, what is the probability that it was at bar C?

$$\mathbb{P}(C \mid N^c) = \frac{\mathbb{P}(C \cap N^c)}{\mathbb{P}(N^c)}$$

$$= \frac{\mathbb{P}(N^c \mid C)\mathbb{P}(C)}{\mathbb{P}(N^c)}$$

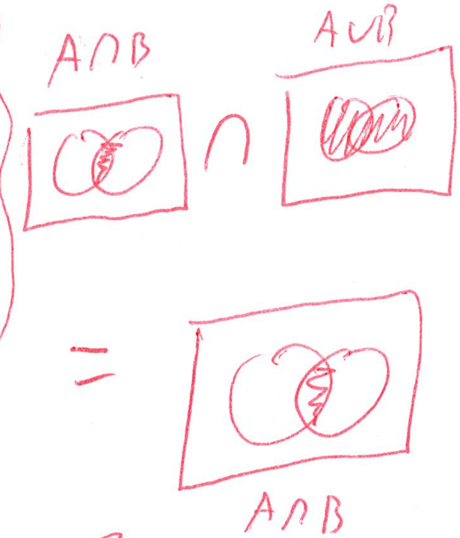
$$= \frac{(1 - .85)\frac{2}{7}}{1 - .9414} = \frac{(.15)(\frac{2}{7})}{.0586} = \boxed{.7314}$$

6. Show that if $\mathbb{P}(A) > 0$, then

$$\mathbb{P}(A \cap B \mid A) \geq \mathbb{P}(A \cap B \mid A \cup B).$$

Proof: Call $x = \mathbb{P}(A \cap B \mid A)$

$y = \mathbb{P}(A \cap B \mid A \cup B)$



WTS: $\boxed{x \geq y}$

- Let's compute them separately

$x = \mathbb{P}(A \cap B \mid A)$

$= \dfrac{\mathbb{P}(A \cap B \cap A)}{\mathbb{P}(A)}$, by def

$= \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)} \neq \infty$ since $\mathbb{P}(A) > 0$

$y = \mathbb{P}(A \cap B \mid A \cup B)$

$= \dfrac{\mathbb{P}((A \cap B) \cap (A \cup B))}{\mathbb{P}(A \cup B)}$

$\overset{\text{see diagram}}{=} \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(A \cup B)}$

- Now since $A \subset A \cup B$, then by Proposition 1, $\mathbb{P}(A) \leq \mathbb{P}(A \cup B)$, which means $\dfrac{1}{\mathbb{P}(A)} \geq \dfrac{1}{\mathbb{P}(A \cup B)}$

therefore $x = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)} \geq \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(A \cup B)} = y$,

which is what we wanted to show!!!